Binary Versus Decade Inductive Voltage Divider Comparison and Error Decomposition

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***Abstract*—An automatic Inductive Voltage Divider (IVD) characterization method that can measure linearity by comparing IVD's with different structures is suggested. Structural models are employed to decompose an error vector into components that represent each divider. Initial tests at 400 Hz show that it is possible to assign independent errors due to the binary and decade structures with a 2σ uncertainty of 0.05 parts per million (ppm) at the measured ratio values.**

I. Introduction

THE intention of this paper is to introduce an automatic calibration procedure that will calibrate both the "Standard" and the "Test" IVD at the same time. This approach is possible when the devices that are compared have different internal structures and the error pattern reflects these structures. This permits a unique model to be used for each divider. Using an automatic IVD bridge [1], measurements can be made at hundreds of ratios, so statistical methods can be applied in determining the measurement uncertainty. The term "error vector" will be used to represent the measured difference between the outputs of a binary IVD (BIVD) and a decade IVD (DIVD) for a set of test ratios.

An IVD is an autotransformer whose setting defines the ratio that relates the output voltage to the input voltage. To obtain a variety of ratios, a number of transformers are cascaded using relays. In the case of the BIVD, p transformers of ratio 1/2 are connected in a binary sequence to give 2^p different ratios. The DIVD consists of q cascaded transformers, each having 10 uniformly spaced taps that are connected in a decade sequence to give 10^q ratios.

When an IVD is loaded, the ratio between its output and input signals differs from the turns ratio, and its errors depend on the impedance of the load. For each transformer in the cascaded structure, a less significant transformer is a load (analogous to a least significant bit in a ladder network digital-to-analog converter). To obtain different ratios, appropriate combinations of transformers are used. This means that the error pattern for the full ratio range of the IVD depends on the impedance of the transformers in use. The largest differential errors will occur at ratio steps where the most significant transformers in the cascade are switched. These transitions occur at different ratios in the binary and the decade structures giving different error patterns. This difference in error pattern makes it possible to separate the error contributions from each device.

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II. Linear Error Model

To decompose the error vector into decade and binary components, a linear error model is employed [2] and [3]. In matrix form this model is given by

$$\mathbf{y} = \mathbf{A}\mathbf{x}$$

where $\mathbf{y}$ is an $m \times 1$ vector that contains the measured errors, $\mathbf{A}$ is an $m \times n$ model matrix, and $\mathbf{x}$ is an $n \times 1$ parameter coefficient vector. The rows of model matrix $\mathbf{A}$ correspond to the different ratios measured, i.e., the test points. Typically, m is considerably larger than n, producing an overdetermined system, which reduces the influence of the random measurement noise, and provides redundancy for detecting model errors.

The model matrix $\mathbf{A}$ is divided into three sections

$$\mathbf{A} = [\mathbf{b} \mid \mathbf{d} \mid \mathbf{s}].$$

The first two sections are the binary $\mathbf{b}(m \times n_b)$ and the decade $\mathbf{d}(m \times n_d)$ models. The third section $\mathbf{s}(m \times n_s)$ is the system model, which consists of vectors that represent the behavior of the measurement system, i.e., offset and gain. This partition separates the parameter coefficients into three groups, with each coefficient representing the error contribution associated with the model vectors. The next two sections of this paper describe the construction of the binary and decade parts of the model matrix $\mathbf{A}$.

To estimate the parameter coefficients $\bar{\mathbf{x}}$, the following least-squares equation is used

$$\bar{\mathbf{x}} = (\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\mathbf{y}.$$

The estimated values of $\bar{\mathbf{x}}$ are used to compare the predicted to the measured errors. The residual error $\mathbf{r}$ is given by

$$\mathbf{r} = \mathbf{y} - \mathbf{A}\bar{\mathbf{x}}.$$

The residual is used to evaluate the model and to estimate the uncertainty of the predicted errors.

Once the parameter coefficients are estimated, it is possible to calculate predictions for the binary, decade, and system errors: $\mathbf{pe_b}, \mathbf{pe_d}$, and $\mathbf{pe_s}$, respectively, by setting the appro-

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14. ABSTRACT
An automatic Inductive Voltage Divider (IM) characterization method that can measure linearity by comparing WD?s with differeut structures is suggested. Structural models are employed to decompose an error vector into components that represent each divider. Initial tests at 400 Hz show that it is possible to assign independent errors due to the binary and decade structures with a 27 uncertainty of 0.05 parts per million (ppm) at the measured ratio values.

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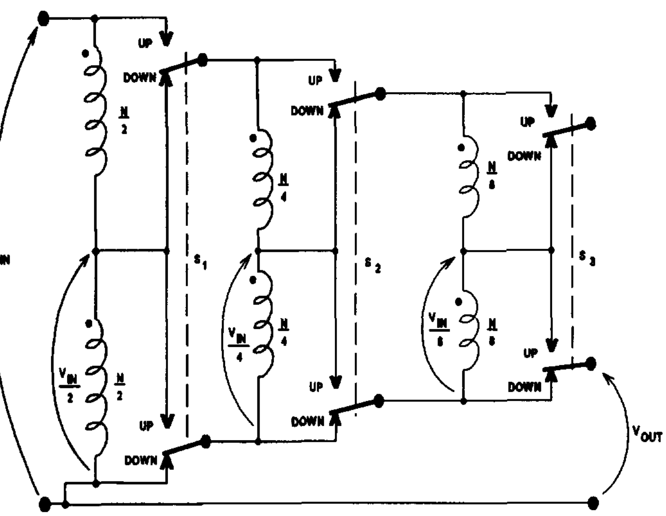

Fig. 1. A 3-bit BIVD structure. S_1, S_2, and S_3 are double-pole (labeled with dashed line), two-position (up, down) switches for the first, second, and third bit, respectively. N is the number of turns. V_{IN} is the input voltage. V_{OUT} is the output voltage.

TABLE I
AN EXAMPLE OF 3–BIT BINARY SETTINGS

Switch position with model vectors	NOMINAL RATIOS							
	0	0.125	0.25	0.375	0.5	0.625	0.75	0.875
S_1 V_1	down	down	down	down	up	up	up	up
S_2 V_2	down	down	up	up	down	down	up	up
S_3 V_3	down	up	down	up	down	up	down	up

Switch positions with associated model vectors (V_1, V_2 AND V_3) for all possible nominal ratios for 3-bit BIVD. Nominal ratio is equal to the ratio of V_{OUT} over V_{IN}.

priate sections of the model matrix $\mathbf{A}$ to $\mathbf{0}$

$$\mathbf{pe_b} = [\mathbf{b} \mid \mathbf{0} \mid \mathbf{0}]\bar{\mathbf{x}}$$
$$\mathbf{pe_d} = [\mathbf{0} \mid \mathbf{d} \mid \mathbf{0}]\bar{\mathbf{x}}$$
$$\mathbf{pe_s} = [\mathbf{0} \mid \mathbf{0} \mid \mathbf{s}]\bar{\mathbf{x}}.$$

Smaller residuals indicate better predictions of $\mathbf{pe_b}$, $\mathbf{pe_d}$, and $\mathbf{pe_s}$. For this reason it is very important to develop a model that will extract as much structure as possible from the measured data.

III. BINARY REPRESENTATION

The BIVD used for this analysis consists of 30 transformers connected in such a way that it is possible to obtain 2^{30} different ratios in the range between 0 and 1.

The basic structure of a 3-bit BIVD is given in Fig. 1. The output voltage is determined by the switch positions, which are shown for all eight possible ratios in Table I. Corresponding model vectors based on this structure are called independent binary switch functions and are used to model the error pattern of the BIVD.

Using this vector representation, it is also possible to extract the error associated with the interaction of two switches. This kind of error is called a "multibit" error. The exclusive-or of two independent switch functions gives the multibit function that is used to describe the interaction of the two independent switches. The number of these functions increases exponentially with the number of bits included in the analysis. Only the multibit errors that are significant have been included in the model.

IV. DECADE REPRESENTATION

The DIVD used in this analysis consists of seven transformers with 10 uniformly spaced taps, which can be arranged to form 10^7 ratios in decade steps. Decade switch functions are used to model the error structure of the DIVD.

The structure of a two-decade DIVD is given in Fig. 2. Each decade is controlled with a switch. The switches in each decade are ganged so that one and only one switch is closed. Fig. 3 shows the decade switch functions that result from this switch structure.

The proposed decade model consists of switch functions that represent the digits independently. Multidecade interactions are approximated by an analytic function described later.

V. ERROR DECOMPOSITION METHOD APPLIED TO THE BIVD VERSUS DIVD COMPARISON

During a comparison, both dividers are set to selected ratios. The full test set would have 2^{30} test points; however, a set of only several hundred random test points was used to uniformly cover the full range of ratios between 0 and 1. Because of the structure of two dividers, they cannot be set at exactly the same ratio. The DIVD has a resolution of 10^{-7}, and the BIVD has a resolution of approximately 10^{-9}, so the difference in set ratios can be as large as 0.5×10^{-9}. This error is not

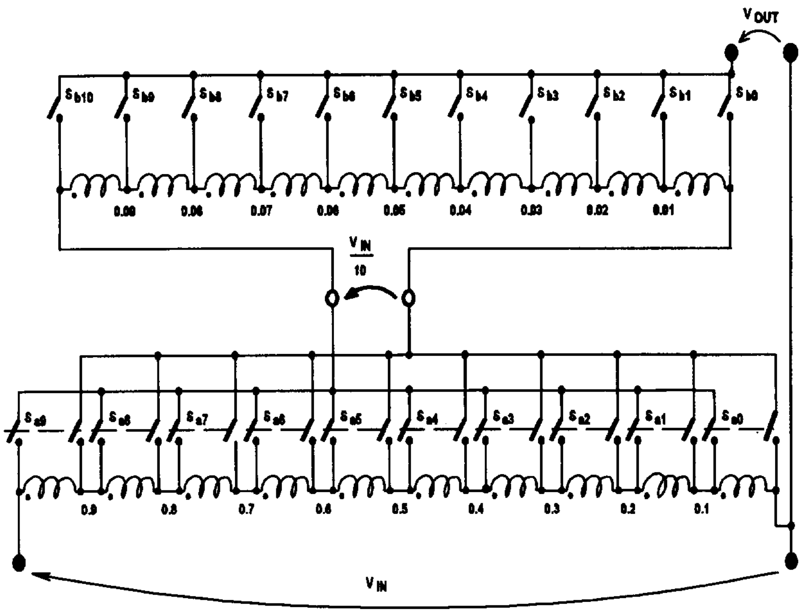

Fig. 2. Two-decade DIVD structure. $S_{a0}, S_{a1}, \ldots, S_{a9}$ are ganged, double-pole (labeled with dashed line), two-position (open, closed) switches used for dials $0, 1, \ldots, 9$ on the first decade. $S_{b0}, S_{b1}, \ldots, S_{b9}$ are single-pole, two-position switches used for dials on the second decade. S_{b10} and S_{a9} are used to obtain a nominal ratio equal to 1.

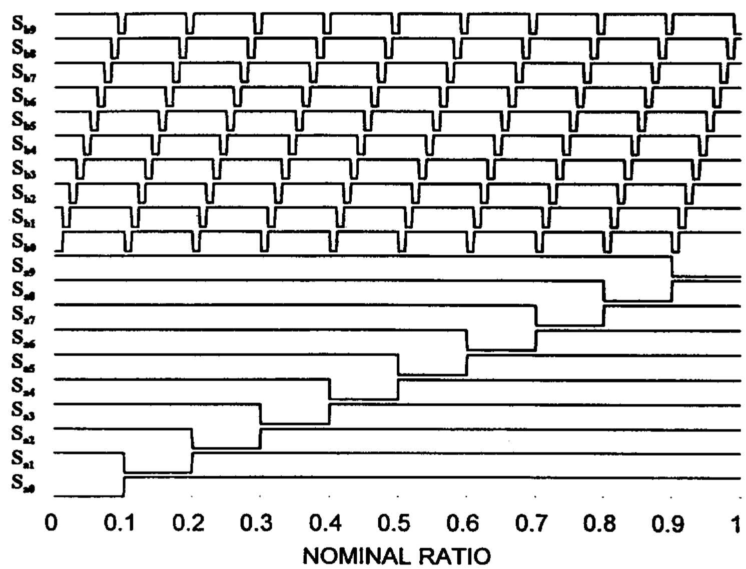

Fig. 3. Model vectors for a two-decade DIVD obtained from switch positions shown on Fig. 2. When a switch is closed the vector has a low value.

significant because the statistically estimated normalized noise level is 0.005 ppm (5×10^{-9}).

The model used in the error decomposition method has binary, decade, and system components. Each of these structures is represented with vectors that span the appropriate vector space. Separately, each basis is orthonormal. But the combined basis, called the model basis, is not orthogonal and may contain linearly dependent vectors. Several techniques, including selection of test ratios, were used to eliminate all linear dependence between the vectors [6].

In addition to selecting test ratios that give good separation of the model components, another technique was employed

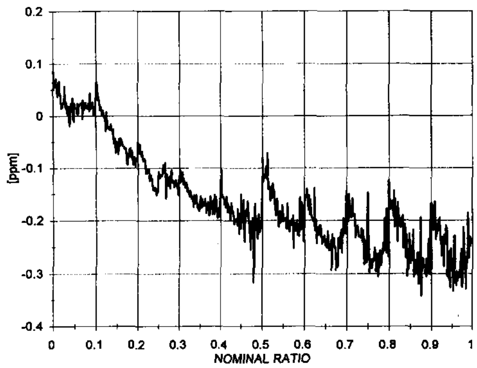

Fig. 4. Measured difference between BIVD and DIVD.

to increase the separation between the binary and decade components. Extra test points for the binary transitions of the three most significant bits were added to the test set. Those transitions occur at the ratios 0.125, 0.25, 0.375, 0.5, 0.625, 0.75, and 0.875. Two measurements were made for each transition, leaving the setting on the DIVD on the nominal value while the BIVD was switched from the nominal setting to the setting that is one least significant bit less than nominal. In generating model matrix **A** for these test points, the two subsequent rows have the same decade elements, but almost totally different binary elements, making a clear distinction between the binary and decade components of the error.

In generating the binary model, independent binary switch functions were used to represent the binary structure, and a multiswitch function that represents the interaction between the first and second bits was added. Decade switch functions were used to represent the decade structure where vectors that represent the 0 digit were omitted in order to satisfy the linear independence requirement. The errors associated with digits 0 through 9 of each decade are linearly dependent upon either the gain of the divider or the vectors of the next most significant decade. Since the gain is considered a system component, these redundancies were eliminated by removing the digit 0 vectors from each decade. Most of the measurements show that the errors of the DIVD are small for the ratios that include digit 0 in the first decade. A constant value vector was added to represent a system offset. A third-degree polynomial curve (S shape) is derived as an error pattern for the influence of the interwinding impedances in the first decade of the DIVD [4], and a vector was added to represent this behavior. Also, a study of the influence of the load impedance when transformers are cascaded [5] showed that the capacitive coupling is predominant. Three vectors were added to model this effect. The vector that represents the output resistance of the DIVD is added to the model to cover the influence of the residual current when the balance is not perfect.

VI. Results

The decomposition scheme proposed allows the separation of the measurement (Fig. 4) into binary (Fig. 5) and decade (Fig. 6) error predictions, and residual error (Fig. 7). Using

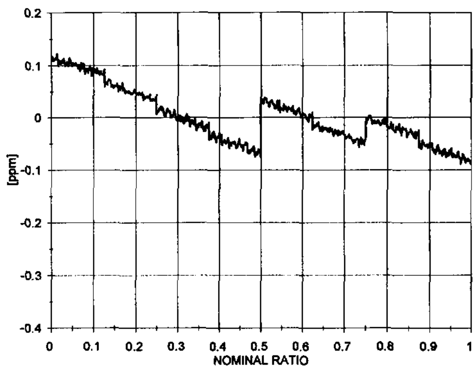

Fig. 5. Binary error predictions.

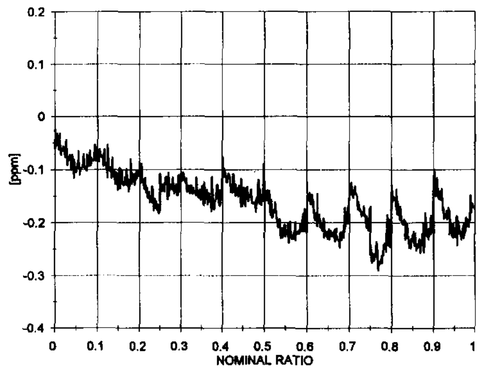

Fig. 6. Decade error predictions.

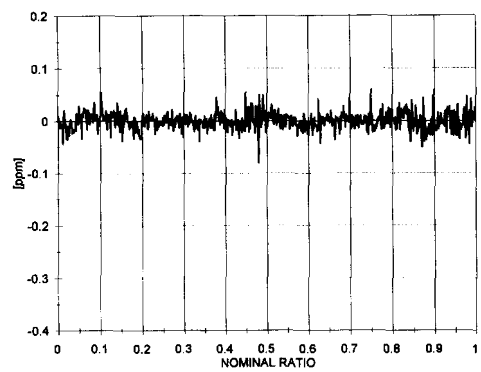

Fig. 7. Residual errors.

the extra vectors mentioned above, it was possible to reduce the rms value of the residuals to 0.017 ppm for a 400 Hz comparison. The rms value of the residuals and noise level combined give a total 2σ uncertainty less than 0.05 ppm. The model consisted of 99 vectors for 680 measured ratios, and the error predictions and residual errors shown in Figs. 5, 6, and 7 were generated using the tested ratios.

Another test was performed where the set of 680 measured ratios was divided into a "model" set and a "validation" set.

The model set was used to generate parameter coefficients for the binary and decade structures. The calculated parameter coefficients were applied to predict the errors for the ratios in the validation set. The standard deviation of the difference between the measurements and predictions was 0.022 ppm. This additional test gives a more accurate predictor of the measurement method.

The completeness of the model is defined as the degree to which a model can describe the measured response of any system for which the model is intended. A model that is complete should produce randomly distributed residuals with an rms value equal to the measurement noise.

The rms of the residuals obtained in the above example is very good with reference to the desired calibration accuracy. Since the rms value is roughly three times the noise level and the residuals show a structure, a more complete model is possible.

More sophisticated analysis of the interactions within the binary and the decade structure is necessary to obtain new multibit or multidecade vector representations. Care must be exercised to assure that the errors are properly assigned to either the binary or decade device.

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REFERENCES

[1] S. Avramov, N. M. Oldham, D. G. Jarrett and B. C. Waltrip, "Automatic inductive voltage divider bridge for operation from 10 Hz to 100 kHz," *IEEE Trans. on I&M*, Vol. 42, no. 2, pp. 131–135, Apr. 1993.

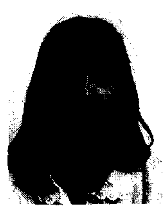
[2] G. N. Stenbakken and T. M. Souders, "Linear error modeling of analog and mixed-signal devices," *Proc. 1991 International Test Conference*, IEEE Computer Society Press, Sept. 1991.

[3] T. M. Souders and G. N. Stenbakken, "A comprehensive approach for modeling and testing analog and mixed-signal devices," *Proc. on International Test Conference 1990*, Nov. 1990, pp. 169–176.

[4] T. L. Zapf, C. H. Chinburg and H. K. Wolf, "Inductive voltage dividers with calculable relative corrections," *IEEE Trans. on I&M*, Vol. 12, pp. 80–85, Sept. 1963.

[5] K. Grohmann, "Error determination of inductive voltage dividers with nondecade ratio settings," *IEEE Trans. on I&M*, Vol. 29, no. 4, pp. 496–501, Dec. 1980.

[6] S. Avramov, "Voltage ratio measurements using inductive voltage dividers," *Ph.D. Thesis*, University of Maryland, College Park, MD, USA.



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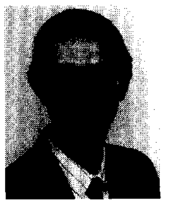

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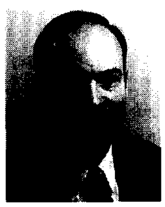

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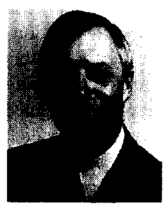

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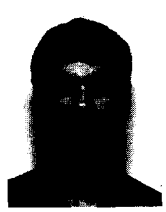

Robert W. Gammon was born and raised in the Washington, D.C. area. He was graduated with an A.B. in physics from Johns Hopkins in 1961, an M.S. from Cal Tech in 1963 and a Ph.D. from Johns Hopkins in 1967. His thesis work was on laser light scattering from soft modes in ferroelectric TGS crystals.

Dr. Gammon's first faculty position was in physics at Catholic University from 1968 to 1972. Since 1972 he has been at the Institute for Physical Science and Technology at the University of Maryland. His research interests have been broadly based on the use of laser light scattering spectroscopy to study materials approaching phase transitions or glass transitions. A particular specialty has been the use of multi-passed Fabry-Perot spectrometers to measure Brillouin spectra from acoustic models.